Propulsion Requirements for Drag-Free Satellites

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Nomenclature

a_x, a_y, a_z = sinusoidal perturbing acceleration along x, y, and z
d = thermal displacement of pickoff
e,i,n = orbit eccentricity, inclination, and mean motion
P = spin period, $2\pi/\omega_s$ sec
p = perturbation frequency
t = time, sec
Δt = thrustor on-time, sec
x,y,z = satellite displacements along the vertical, intrack, and orbit normal
ΔV = total impulse/satellite mass
τ = time delay in thrustor response, sec
ω_s = spin rate, rad/sec
ω_x = control system crossover frequency

Introduction

ON orbit, satellites are disturbed by very small forces. In some missions these are important and prevent a satellite from following a purely gravitational orbit. Lange[1] proposed the solution for making the satellite drag-free by measuring the relative position of the satellite with respect to an internal shielded, unsupported proof mass or ball and using the error signal to actuate thrustors, the satellite can be forced to follow the purely gravitational orbit of the proof mass. Thrust, impulse, and bandwidth requirements for drag-free satellites vary by many orders of magnitude, depending on the mission. This Note discusses propulsion requirements for four missions and some considerations peculiar to drag-free satellites.

Missions

Aeronomy

Absorption of solar energy in the altitude band between 100 and 250 km affects the atmospheric properties. More accurate measurements of density, temperature, and composition are needed to improve atmospheric models.[2] A drag-free-satellite measurement of density in this region has been proposed, and a comparison of this technique with accelerometer measurements has been made.[3] The drag-free satellite is superior in launch requirements for apogees lower than approximately 500 km. The satellite should be spherical to facilitate the calculations of the drag coefficient. A lifetime of at least a week is required to insure that one solar storm will occur during the satellite's life, so that correlation of the atmospheric response to solar activity can be made. Perigee altitudes in the range of 120–150 km appear practical, but the orbit must be eccentric to obtain the required life—apogee should be at least 400–500 km. More information is obtained about the structure of the atmosphere if the spatial resolution of atmospheric density measurements is finer than an atmospheric scale height along the orbit. The control force will give this information if measured with adequate bandwidth, because the control must balance the disturbance if the satellite follows a purely gravitational orbit. Control-

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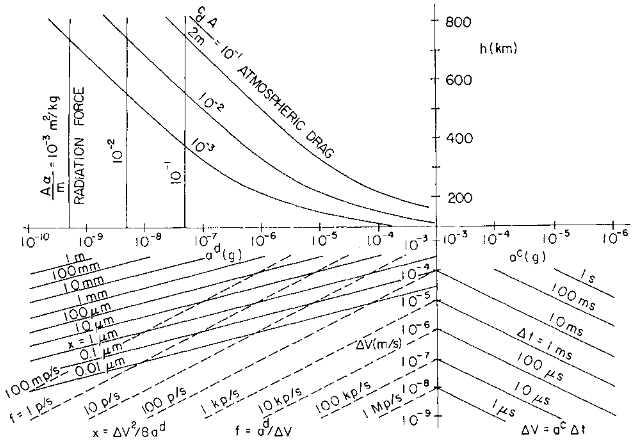

Fig. 1 Performance parameters for pulsed drag-free control systems as a function of altitude. The satellite disturbance is estimated as a function of altitude in the second quadrant; the control acceleration and on-time may be chosen in the fourth quadrant to give the required pulse frequency f to insure adequate measurement bandwidth without excessive motion x where f and x are shown in the third quadrant.

force measurements should be made to an accuracy of 1%, and they should resolve at least 1-sec variations near perigee and approximately 10-sec variations near apogee. This requires on the order of 5 samples/sec for pulse control near perigee. The bandwidth of the control system is further determined by the attitude motion. Allowing the satellite to tumble randomly, ω_s probably will not exceed 1 rad/sec. Thus, if pulse control is used, minimum thrustor on-times of ~20–30 msec are acceptable.

Figure 1 shows the parameters that are significant in sizing a pulsed control system. In the second quadrant, the atmospheric drag and radiation force are plotted vs altitude parametrically as a function of their appropriate surface-to-mass ratio parameters. In the fourth quadrant, the abscissa indicates the control acceleration. This may be determined by an acceptable settling time, or by the maximum control force required at perigee. The thrustor on-time then determines the ΔV of the satellite for each pulse. Assuming the external disturbing force is constant between control actions, the relative motion is parabolic with time. The third quadrant shows the amplitude of these parabolas as a function of the disturbing acceleration and the ΔV. At some low level of disturbing acceleration, the amplitude of this motion will exceed the deadband of the control system, and propellant will be wasted. Hence, the impulse bit size should be small to prevent two-sided limit cycling. On the other hand, impulse bit size should be large to minimize the total number of control system actuations during the satellite lifetime to increase reliability. The pulse repetition rates are also given, and with this information, an appropriate deadband and impulse bit size can be chosen which results in adequate bandwidth for making the measurement and adequate deadband to prevent two-sided limit cycling. The extremely small bit size possible with some electric propulsion systems would ease the requirements for adequate band width over a typically three- to four-orders-of-magnitude range of average thrust required from perigee to apogee for this mission. In a cold-gas propulsion system, the time between firings at apogee is so long that it is necessary to differentiate the relative position signal to obtain adequate band width in measuring the external disturbances.

Geodesy

The gross structure of the Earth's gravitational field has been determined by tracking conventional satellites. However, determinations of the higher fixed tesseral harmonics, and the extremely small temporal variations due to the lunar and solar tides on the Earth have been limited by the small but significant disturbance forces. The higher tesseral harmonics have a greater effect on the satellite when it is at low altitude. A nearly circular orbit with an altitude as low as 200 km and a lifetime of two months or more (geodesy 1, Table 1)[4] would be desirable, but a 1000-km orbit and 2-yr life (geodesy 3, Table 1) can be used at some loss in accuracy but with less demanding propulsion requirements. With a low, nearly circular orbit, differentiation of a Doppler tracking signal yields gravitational acceleration directly. An alternative technique has been proposed in which a highly eccentric orbit (geodesy 2, Table 1) is resonant with the Earth's rotation. All harmonics that are longitudinally in resonance with the orbit excite long-period variations in the orbit. The eccentricity, due to the rotation line of apsides, however, separates these effects, so that with a single orbit, many of the higher tesseral harmonics can be determined by the observation of the long-period perturbations they produce. A low perigee is required to obtain sensitivity as previously stated. The principal temporal effect that is of interest is due to the tidal bulge, which is predominantly a second harmonic in the Earth's gravitational field. Therefore, the effect is not as sensitive to the altitude selected, but a longer time is required for the effect to build up to a measurable perturbation. An altitude of 1000 km, with $70° \leq i \leq 80°$ (geodesy 3, Table 1), has been recommended for this mission.

Although the proof mass may be shielded from external disturbing forces, various interaction forces between the satellite and the proof mass may cause very minute disturbances. The principal interaction forces are mass attraction and pickoff forces caused by, for example, a capacitive bridge. It is particularly important to the geodesy mission that disturbances on the proof mass not cause long-period variations which might be indistinguishable from those due to some of the resonating tesserals. The magnitudes of the effect of variations in these disturbing forces on the satellite ephemeris can be calculated for sinusoidal forces which are slow compared with an orbital period as,

$$x = a_x/n^2 + 2a_y/np \tag{1}$$

$$y = 2a_x/np + 3a_y/p^2 \quad z = a_z/n^2 \tag{2}$$

The cross-track (a_z) and vertical (a_x) disturbances have little effect, but the in-track effects (a_y) are large, because they produce variations in the orbit energy. Assuming that one should design the satellite so the in-track disturbances are smaller than the state-of-the-art tracking error, one requires $a_y \leq 10^{-11}g$ to prevent long-period variations with a period of one week from producing a 1-m amplitude perturbation.

Navigation

The ability to predict the ephemeris of a navigation satellite accurately is of great importance. In fact, if a satellite follows an orbit different than the one predicted, a control system can be used which perturbs the proof mass so it follows the desired orbit. The errors in the forcing capability or disturbances of the satellite on the proof mass produce long-period effects on the orbit, particularly in the in-track direction where these disturbances affect the energy of the orbit. For a constant a_y, the displacement is

$$y = -3a_y t^2/2 \tag{3}$$

Thus, a constant a_y of $10^{-11}g$ will produce an ephemeris error of 10 m after three days. This extreme sensitivity to proof-mass disturbance should not be confused with the stability of the thrust level of the propulsion system. Large uncertainties in the thrustors are tolerable because they appear in the forward loop of the translation control system.

Instrument carrier

Gyroscopes are limited in their performance on the ground by the accuracy with which the support force can be aligned through the mass center. On orbit, the required supporting force is smaller by perhaps six orders of magnitude. To remove this small remaining force completely a gyroscope may be used as the reference proof mass in a drag-free satellite. To determine the accuracy with which a gyroscope operates, however, requires an extremely precise attitude-control system, so that the orientation of the gyroscope may be compared with the line of sight to a star. In some missions, this can be accomplished by spinning the satellite. An ω_s of $\sim$ 1 rad/sec appears feasible, though a higher ω_s would simplify the attitude-control-system design. The propulsion system must, therefore, be compatible with this ω_s and be capable of a) responding quickly ($\tau \ll P = 2\pi/\omega_s$) and b) turning on and off quickly ($\Delta t \ll P$).

Summary of Propulsion Requirements

Table 1 gives approximate values for thrust and impulse requirements for preliminary design purposes, but each value may change by an order of magnitude upon detailed evaluation of a specific mission. Some additional considerations are discussed in this section.

Mass attraction

The satellite must be built so that its parts are distributed uniformly about its proof mass. An uncertainty of 1 g at 2.5 cm would account for an entire error budget of 10^{-11} g, and 1 g at 1 cm would usurp a mass attraction gradient budget of $10^{-11}g$/mm.[5] As indicated by Eqs. (1–3), the effect on the

Table 1 Mission dependence of drag-free satellite requirements

		Geodesy			Navigation		Instrument testing
	Aeroeomy	1	2	3	1	2	
Orbit e	$\sim$0.025	0	<1	0	0		0
i, deg	90	90	various	70 – 80	90		90
Disturbance, g	$10^{-7} - 10^{-3}$	$5 \times 10^{-5} - 5 \times 10^{-4}$	$10^{-8} - 10^{-4}$	$10^{-8} - 10^{-7}$	$10^{-8} - 10^{-7}$		$10^{-8} - 10^{-7}$
Life, yr	0.03	0.2	2	2	7		1
Minimum ΔV, m/sec	100	2500–10,000	100	6	40 – 400		6
Spin rate, rad/sec	random	$>10^{-1}$	$>10^{-1}$	$>10^{-2}$	orbit rate	$\sim10^{-4}$ or 10^{-2}	$\sim$1
Mass center tolerance, m	10^{-3}	10^{-4}	10^{-4}	10^{-3}	10^{-2}	10^{-3}	3×10^{-5}
Mass attraction requirements, g	10^{-7}	10^{-9}	10^{-9}	10^{-10}	10^{-11}	10^{-10}	10^{-8}
Allowable time delay in thruster response, sec	0.01	0.05	0.05	0.5	1	0.5	0.01

orbit is dependent on the strength and frequency of disturbances.

Propellant storage is a critical consideration. A gaseous propellant fills its storage volume uniformly if isothermal, and intolerable temperature gradients can be avoided. Liquids and solid propellants present a problem, but the changes and uncertainties may be acceptable if they are relatively constant and the satellite is spun. Two toroidal propellant tanks may be used to a) locate the mass center of the propellant at the proof mass to avoid propellant waste due to the gradient in the gravity field and to prevent coupling with the rotational dynamics of the satellite and b) minimize the gradient in the mass attraction field.

When the satellite's orientation is fixed relative to the local orbit coordinates, a mass attraction force or other body-fixed disturbance on the proof mass can produce a significant disturbance. If, on the other hand, a satellite is rotated, the average effect of these disturbances can be made smaller by an order of magnitude or more. Because of the cost required to reach a $10^{-11}g$ uncertainty level in the mass attraction of a proof mass, the rotation of a satellite about either the vertical or the normal to the orbit plane is highly desirable. This, of course, places requirements on the speed of response and the acceptable time delays of the control system, and in particular the thrustors.

Initial settling, control bandwidth and time delay

The dynamics of the motions of the satellite with respect to the proof mass is straightforward if both have the same gravity or are in a field-free space. This approximation is valid if the acquisition and settling takes place in a time which corresponds to a small change in central angle of the orbit; the orbital coupling of the horizontal and vertical motions can be neglected. If 200 sec is taken as a nominal settling time, the bandwidth of a linear system would be 3×10^{-2} rad/sec. In a nonlinear controller, some equivalent measure could be used.

If we arbitrarily require that the time delay τ reduces the phase margin less than 0.1 rad near the crossover frequency ω_x, then $\tau < 0.1/\omega_x$, and τ should be less than 3 sec for the settling behavior. Similarly, if we require $\tau < 0.1/\omega_s$ for spinning satellites, and ω_s could be as high as 1 rad/sec, we require $\tau < 0.1$ sec.

Disturbances

Thermal distortions in the satellite may change the effective null point of the pickoff by a discrete amount each time the satellite enters and leaves the sunlight. It is important to keep the impulse required to respond to this change small compared with the total impulse required to counteract the external disturbing forces. Regardless of the control mechanization involved, the impulse requirement is of the order of $\Delta V \approx 4d\omega_x$, where d is the amplitude of the sudden change in the null location. There are variations of a factor of 2 or 3 for different control mechanizations. Assuming a settling time of 100 sec, the allowable thermal distortion of the satellite to maintain $\Delta V < 4 \times 10^{-5}$ m/sec would be a displacement of 1 mm.

The total impulse required to counteract an external impulse or step change in the external force should be independent of bandwidth as long as the mechanization is adequately damped.

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Life Test Summary and High-Vacuum Tests of 10-mlb Resistojets

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THIS Note summarily describes the results of an 8,000-hr life test of six 10-mlb-thrust, high-temperature (2400°K) evacuated-concentric-tubes type resistojets and presents details of a high-vacuum test that was conducted on one of the resistojets to document the effect of cell pressure on thrust. Reference 1 describes the evacuated-concentric-tubes resistojet concept. This resistojet, in the ten-mlb-thrust size, is being developed primarily for use on manned space stations for drag make-up and for attitude control in conjunction with control-moment gyros. This Note describes resistojets with heater elements made of rhenium to be used with hydrogen and ammonia propellants. Reference 1 describes the development of related resistojets for use with biowaste propellants.

Life Test Summary

The life test was comprised of the simultaneous testing of a cluster of six resistojets; four were operated with NH_3 and two with H_2. A 50/50 duty cycle-one cycle per hour, with the thrusters in two groups, two NH_3 and one H_2 thruster per group, was used. The four NH_3 thrusters were cycled in excess of 8000 hr. Anomalies occurred with the H_2 thrusters (S-1 and S-2) which are attributed to experimental fabrication techniques and not associated with the hydrogen propellant; thruster S-1 developed a leak and was left in the life test to obtain temperature data, and thruster S-2 was removed from the life test after 1426 hr. Another thruster, B-2, was substituted for S-2 and successfully completed 6023 hr of operation with hydrogen, at which time the life test was termi-

Table 1 Nominal values for life-test thrusters

Thruster serial no.	S-1	S-2	B-2	S-3	S-4	S-5	S-6
Propellant	H_2	H_2	H_2	NH_3	NH_3	NH_3	NH_3
Test duration, hr	7858	1426	6023	8048	8152	8052	8134
I_{sp}, sec (estimated for space)	660	670	670	320	320	320	320
Electric power, w	258	220	222	131	189	145	145
Thrust, mlb	11.7	10.0	10.5	9.2	11.9	10.6	10.5

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